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THE USE OF HIGHER RESONANT MODES IN MEASURING THE DIELECTRIC CONSTANT OF DIELECTRIC RESONATORS*

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Abstract

In the measurement procedure known as Courtney method, the dielectric constant of a cylindrical dielectric resonator is determined by measurement of the resonant frequency of the TE_{011} mode. The reliability and accuracy of this method can be improved by also utilizing higher resonant modes. A procedure for identification of the modes is described, and some example results for measured dielectric constant are presented.

Introduction

For accurate design of microwave filters and oscillators employing cylindrical dielectric resonators, the relative permittivity of the resonator material must be known with certainty. The experimental procedure commonly in use for determining the dielectric constant was first described by Hakki and Coleman [1], and later analyzed and popularized by Courtney [2]. It is simply known under the name "Courtney Method."

The Courtney procedure consists of placing the dielectric resonator of circular cross-section between two parallel metal plates, as shown in Figure 1, and measuring the resonant frequency of mode TE_{011}. Since a closed-form analytical expression for the resonant frequency in this operating environment is available [2], it is possible to compute the value of dielectric constant from knowledge of the resonant frequency and the physical dimensions of the resonator.

When the actual measurement is performed, it is soon realized that a multitude of other resonant modes can be observed in addition to the TE_{011} mode. Hakki and Coleman [1] already suggested that higher modes of the TE_{0np} family can be employed in the procedure. However, difficulty exists in conclusively identifying the modes, because also mode types HEM_{mnp} and TM_{0np} have closely spaced resonances in the frequency range utilized. Therefore, they suggested to construct two, or three, parallel-plate resonators of different size, compute the dielectric constant for all the modes of various indices n and p, and then search for those solutions which have resulted in an identical value of dielectric constant in both (or all three) resonators. Because of such ambiguity in identifying the individual resonant modes, measurements thus far have concentrated on the TE_{011} mode only.

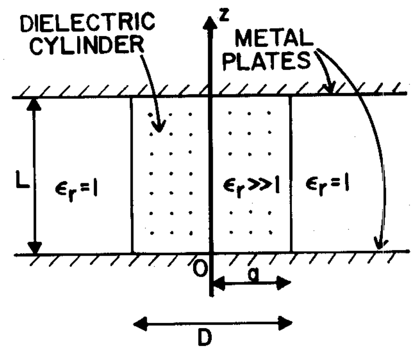

Figure 1. Cylindrical dielectric resonator between parallel metal plates

Modal frequencies and field patterns

The difficulty in identifying the various resonant modes stems partly from the fact that the theoretical frequencies of occurrence must be computed from transcendental eigenvalue equations involving several ordinary and modified Bessel functions [3]. Furthermore, the electric and magnetic field patterns of the modes have not been readily available in the literature. Knowledge of the field patterns is of great importance in deciding what shape and position must be given to devices for coupling to the dielectric resonator - probes, loops, irises, etc.

With the aid of a computer, both of these difficulties can be greatly alleviated.

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Classification of the modes and the numerical evaluation of the eigenvalues has been presented in [3]. Graphical display of many modal field patterns has been given in [4]. These results substantially assist one in anticipating the modal frequencies and the appropriate coupling whereby specific modes of interest may be strongly excited (or suppressed).

Experimental mode identification

Numerous resonances are observed in the course of the measurement procedure. Only upon knowing positively which mode is observed may one trust the results of entering that resonant frequency into the computer program for solution of the transcendental equation. By combining the guidance results of [3] and [4] with an experimental mode identification technique, it is possible to conclusively identify a great number resonances occurring in the Courtney method.

The parallel-plate resonator used in these measurements is shown in Figure 2. The cross-sectional view in Figure 2a shows the cylindrical dielectric resonator of radius a and length L. The two coaxial cables shown have small loops for coupling to the magnetic field of the resonator. The vertical position of the loops can be varied with the use of grounding screws which protrude through each of the two parallel plates. This variation has been found useful in identifying the third subscript p of the resonant mode. In particular, if p=1, the field variation shows one maximum along the z direction while, for p=2, one can observe two maxima, with a minimum at the position where the mode p=1 had a maximum.

The second feature applicable to identifying the modes is given by rotating the receiving loop by 90° about the axis of the coaxial cable. Studying the field patterns of the individual modes in [4], it is possible to conclude whether horizontal or vertical orientation of the coupling loop plane should provide pronounced stronger coupling to the particular mode of interest. It has been observed experimentally that the two perpendicular orientations produce a change in observed mode amplitude of 10-20 dB with regard to the TE and TM modes. In particular, horizontal orientation is preferable for coupling to the TE_{0np} modes, whereas vertical orientation is superior for coupling to the TM_{0np} family. The orientation of probes in Figure 2a is called vertical.

The third indicator for mode identification is azimuthal displacement of the receiving probe. In Figure 2b the top view of the holder can be seen. The receiving loop can be mounted at any one of three different locations, which are marked as 180°, 135°, and 90°. If care is taken to keep the radial distance constant in the course of azimuthal displacement, comparison of the signal amplitudes at the different azimuth angles helps to identify index m of the resonant mode. The circularly symmetric mode (m=0) should display the same signal amplitude for all three positions. A mode with index m=1 should move from a maximum to a minimum over 90° of azimuth change, and modes with index m=2 should do the same within 45° of change.

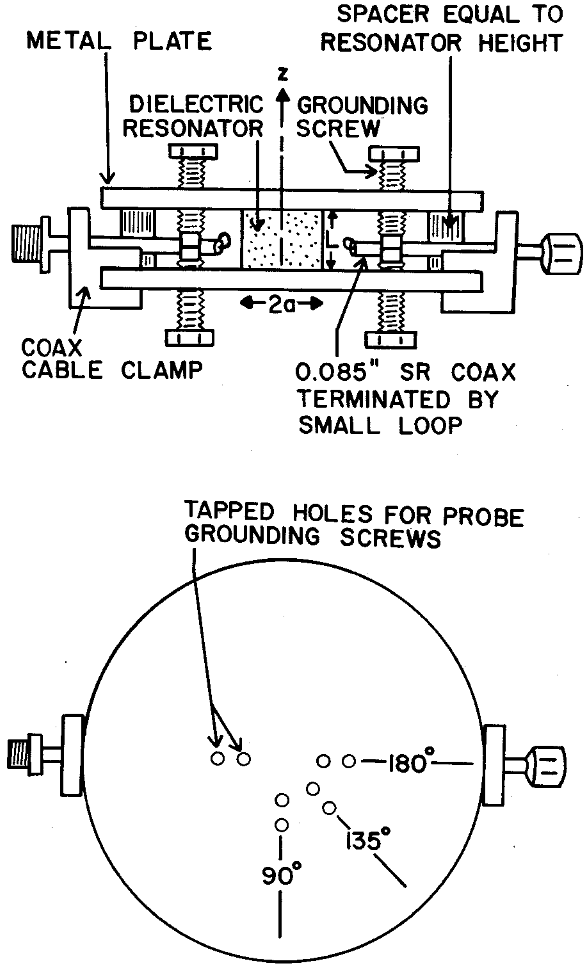

Figure 2. Modified Courtney holder
(a) side view
(b) top view

The results of one mode identification study, with data taken on a Hewlett-Packard 8410B Network Analyzer, are displayed in the mode chart of Figure 3. The horizontal axis places the modes in growing order of frequency, with equal horizontal spacing between mode depictions for clarity of presentation. The vertical axis gives the observed signal level in decibels. The azimuthal positions are indicated by symbols ○ (for 180°), △ (for 135°), and □ (for 90°). For example, each member of the TE_{0np} family of modes, with first subscript m=0, theoretically should have all three observation symbols at the same vertical level. It has been observed experimentally that this condition is manifested by actual values within approximately 3 dB of each other.

The modes of Figure 3 which have been positively identified in the above manner have been indicated by modal names in the figure; the mode at 6.47 GHz has been left unmarked, as its identity is inconclusive.

For the measurement series shown by Figure 3, the transmitting and receiving loops were both horizontally oriented to minimize coupling to the TM modes. Further, the loops were both at z-coordinate L/2, so that p=1 modes were strongly coupled while p=2 modes were suppressed. Although the receiving probe was varied along the z direction to confirm p=1 for the modes shown, data of that nature is omitted from Figure 3 to avoid obscurity. Later, the loops were rotated by 90° to vertical orientation to enhance the modes TM_{011} and TM_{021}, which are also included in Figure 3.

The hybrid, or HEM, modes may be subdivided into two major groups. Since the transverse fields are superpositions of TE and TM parts, the power contained in these respective components may be calculated. Power contained in the TE part (P_{TE}) is segregated from power in the TM part (P_{TM}) due to orthogonality. A hybrid mode with ratio P_{TE} / P_{TM} greater than unity exhibits a predominantly TE character, while TM-like behavior is found associated with those HEM modes with ratio P_{TE} / P_{TM} less than unity.

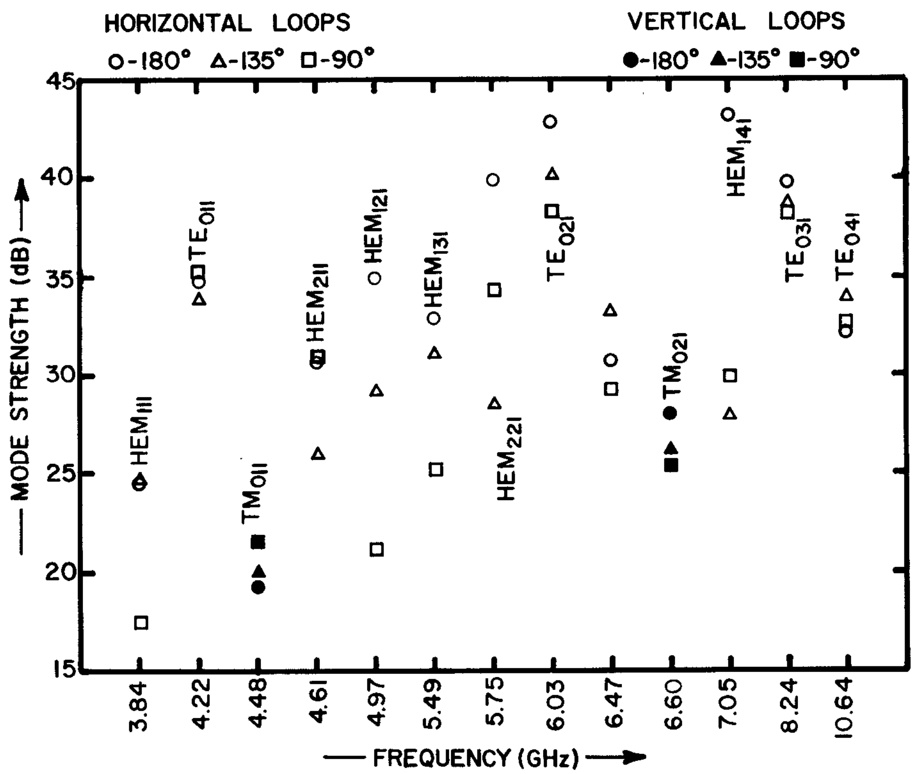

Figure 3. Mode identification chart

Utility of mode types

In addition to ease of locating and identifying the mode TE_{011}, Courtney [2] noted an important attribute of this mode - it is insensitive to an air gap between the dielectric and the metal plates, as the electric field tends to zero at these points. Indeed, all members of the TE_{0np} group share this feature. On the other hand, the TM_{0np} modes are very sensitive to such an air gap. The TM_{0np} modes, hence, are generally undesirable modes for the accurate determination of dielectric constant.

It turns out [3] that the predominantly TE hybrid modes (called quasi-TE modes) are those with an even second mode index (i.e., n=2,4,...) while the predominantly TM (quasi-TM) modes all have an odd second index (i.e., n=1,3,...). Possessing desirable TE-like nature, the quasi-TE modes have a secondary, but definite, utility in the Courtney method when higher resonant modes are employed. The quasi-TM modes, generally, will fall between the quasi-TE and TM types in utility in accurately determining dielectric constant.

Measurement results †

Several identified modes from Figure 3 have been used to determine the dielectric constant of this example dielectric resonator. Eigenvalue equation solutions were obtained on an IBM 4341 computer, with a program using standard IMSL Library functions for the Bessel functions. The results are collected in Table 1.

The sample in this case is a Trans-Tech model D-8515.750.300. The manufacturer's catalog (Dielectric Bulletin No. 41-83) gives a dielectric constant of 37.5 ± 0.56 for this model. Two quality control sheets were included with the several dielectric resonators ordered, one sheet stating relative permittivity to be 35.91 and the other 36.60, without identifying the applicable dielectric resonators.

The weighted means [5] in Table 1 represent the best estimate of a single dielectric constant value, by mode groups, over the stated frequency ranges. Manufacturers' data sheets do not indicate frequency dependence of relative permittivity, and it is taken to be constant over these limited frequency spans. However, experimental data for TE_{0n1} modes do indicate a slight, monotonic decrease in dielectric constant with increasing frequency.

For estimation of experimental accuracy, a numerical variation analysis has been performed, which gives the uncertainty limits cited in Table 1. This analysis includes reasonable error allowances for distance between the metal plates L (±1 mil), resonator diameter D (±1 mil), and measured resonant frequency (±1 MHz). However, it does not include the effect of any air gap between the resonator ends and the metal plates. For the TE modes, the effect of such a gap, believed to be less than 1 mil, is well-approximated by the "L" uncertainty. The problem becomes of progressively greater consequence for the quasi-TE, quasi-TM and, finally, TM mode groups. Unfortunately, a rigorous analysis of air gap effect in those cases is not available. It is for this reason that the uncertainty limits expressed in Table 1 are too small for the quasi-TM and TM modes.

Conclusions

With a reliable methodology for experimentally verifying mode identities, many higher order resonant modes can be incorporated into the traditional Courtney method of measuring dielectric constant. The TE_{0np} family of modes provides excellent results, with multiple results reinforcing and improving the single TE_{011} result obtained to date. The quasi-TE modes (modes with even second subscript) are also useful in this method. Quasi-TM and TM modes are less reliable, and their use is not recommended.

Table 1. EXAMPLE OF MEASUREMENT RESULTS USING HIGHER RESONANT MODES, D = 19.05 mm, L = 7.62 mm

	MODE	F(GHZ)	ε_r
TE Modes	TE_{011}	4.221	35.63
	TE_{021}	6.026	35.48
	TE_{031}	8.241	35.39
	TE_{041}	10.636	35.35
weighted mean		35.42 ± 0.10†	
Quasi-TE Modes	HEM_{121}	4.967	35.39
	HEM_{221}	5.754	35.15
	HEM_{141}	7.047	35.37
weighted mean		35.30 ± 0.13†	
Quasi-TM Modes	HEM_{111}	3.839	34.63
	HEM_{211}	4.608	33.65
	HEM_{131}	5.488	36.63
weighted mean		35.37 ± 1.65†	
TM Modes	TM_{011}	4.478	37.31
	TM_{021}	6.603	36.61
weighted mean		36.84 ± 0.47†	
† - see text.			

References

[1] B.W. Hakki, P.D. Coleman, "A dielectric resonator method of measuring inductive capacities in the millimeter range," IEEE Trans. Microwave Theory and Tech., Vol. MTT-8, pp. 402-410, July 1960.

[2] W.E. Courtney, "Analysis and evaluation of a method of measuring the complex permittivity of microwave insulators," IEEE Trans. Microwave Theory and Tech., Vol. MTT-18, pp. 476-485, August 1970.

[3] D. Kajfez, "Basic principles give understanding of dielectric waveguides and resonators," Microwave Systems News, Vol. 13, pp. 152-161, May 1983.

[4] D. Kajfez, "Modal field patterns in dielectric rod waveguide," Microwave Journal, Vol. 26, pp. 181-192, May 1983.

[5] Phillip R. Bevington, Data Reduction and Error Analysis for the Physical Sciences, McGraw-Hill, 1969, p. 130.